TECHNISCHE UNIVERSITÄT WIEN

Project Page

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Efficient Continuous Group Convolutions for Local SE(3) Equivariance in 3D Point Clouds

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1. Introduction

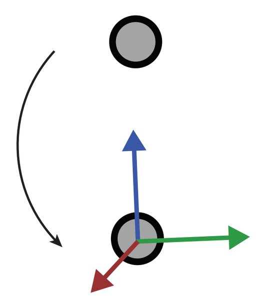
Equivariance: transformation of feature map **commutes** with operator

$$\forall g \in G : \rho^{\mathcal{Y}}(g) \circ \Phi = \Phi \circ \rho^{\mathcal{X}}(g)$$

3D objects can appear in any orientation, introducing significant rotational variability.

Leveraging symmetries beyond translation equivariance enhances weight sharing and **boosts network expressivity** without adding parameters.

2. Group Convolutions

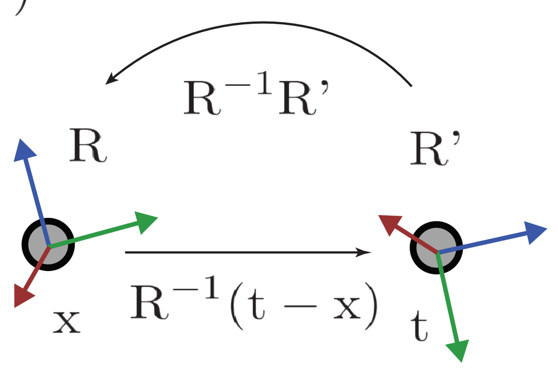


$$\int_{\mathbb{R}^d} f(t)k(t-x)dt$$

Lifting to the group

$$\int_G f(g')k(g^{-1}g')d\mu(g')$$

$$\mathrm{SE}(3) = \mathbb{R}^3 \rtimes \mathrm{SO}(3)$$



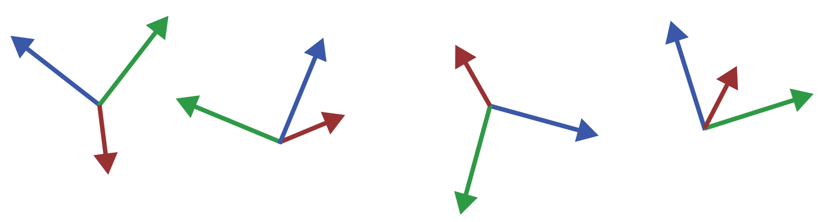
$$\int_{\mathbb{R}^3}\int_{\mathrm{SO}(3)} f(\mathrm{t},\mathrm{R}')k(\mathrm{R}^{-1}(\mathrm{t}-\mathrm{x}),\mathrm{R}^{-1}\mathrm{R}')d\mathrm{t}d\mu(\mathrm{R}')$$

6D convolution

How to define a grid on SO(3)?

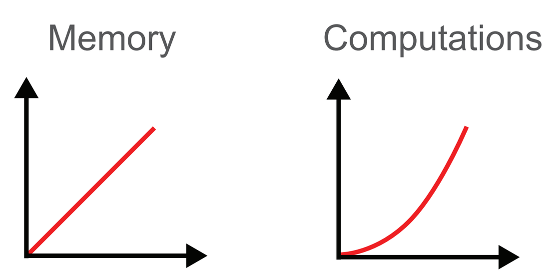
3. Monte Carlo Sampling

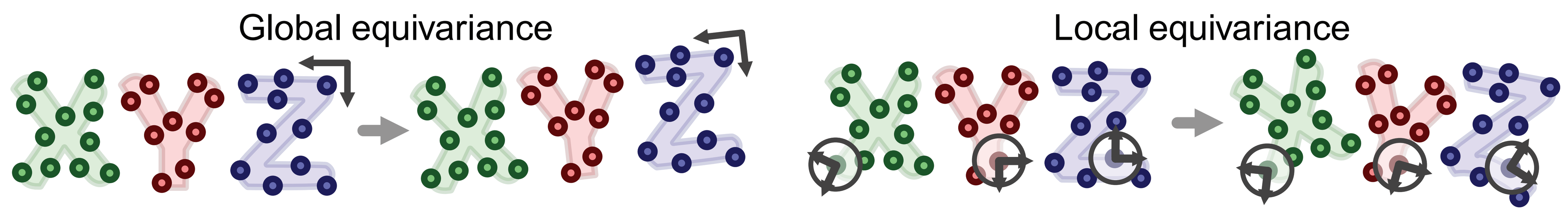
Random grid on SO(3)



$$\sum_j \frac{1}{|H'_j|} \sum_{(\mathrm{t},\,\mathrm{R}')\in H'_j} f(\mathrm{t},\mathrm{R}')k(\mathrm{R}^{-1}(\mathrm{t}-\mathrm{x}),\mathrm{R}^{-1}\mathrm{R}')$$

Trade-off between computational efficiency and integral approximation quality





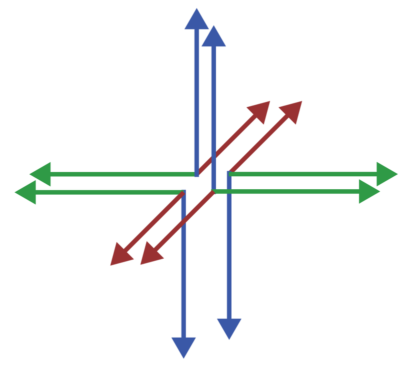
We present an SE(3)-equivariant convolution operator for local continuous feature extraction, avoiding the computational costs of traditional group convolutions.

4. Efficient Group Convolutions

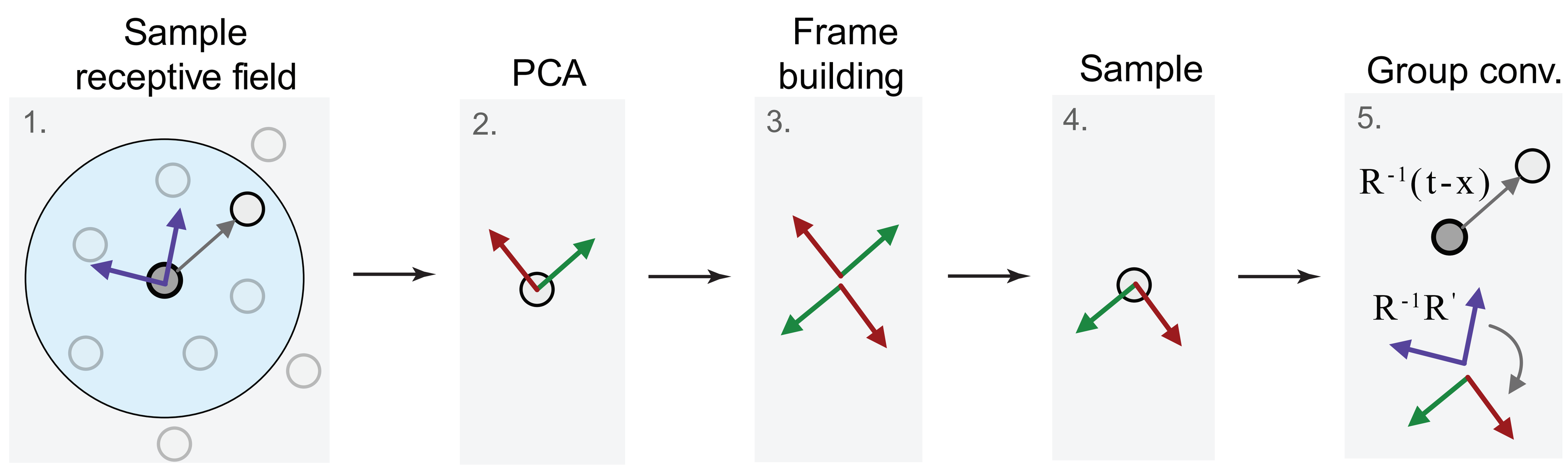
Frame as a point-specific grid on SO(3)

$$\mathcal{F}(x) : \mathbb{R}^3 \rightarrow 2^{\mathrm{SE}(3)} \qquad \mathcal{F}(x) \subset SE(3) \qquad |\mathcal{F}(x)| = 4$$

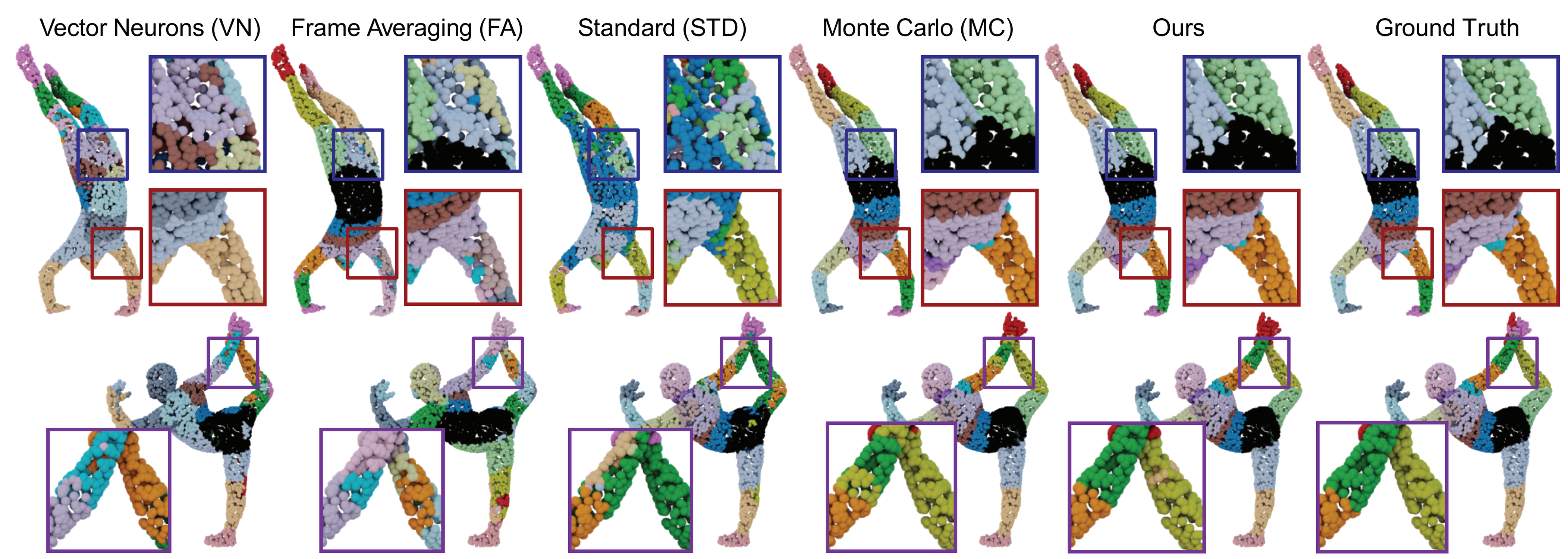
$$\forall g \in G : g\mathcal{F}(x) = \mathcal{F}(gx)$$



$$\sum_j \frac{1}{|\mathcal{F}(x_j)|} \sum_{(\mathrm{t},\,\mathrm{R}')\in\mathcal{F}(x_j)} f(\mathrm{t},\mathrm{R}')k(\mathrm{R}^{-1}(\mathrm{t}-\mathrm{x}),\mathrm{R}^{-1}\mathrm{R}')$$



8. Qualitative Results

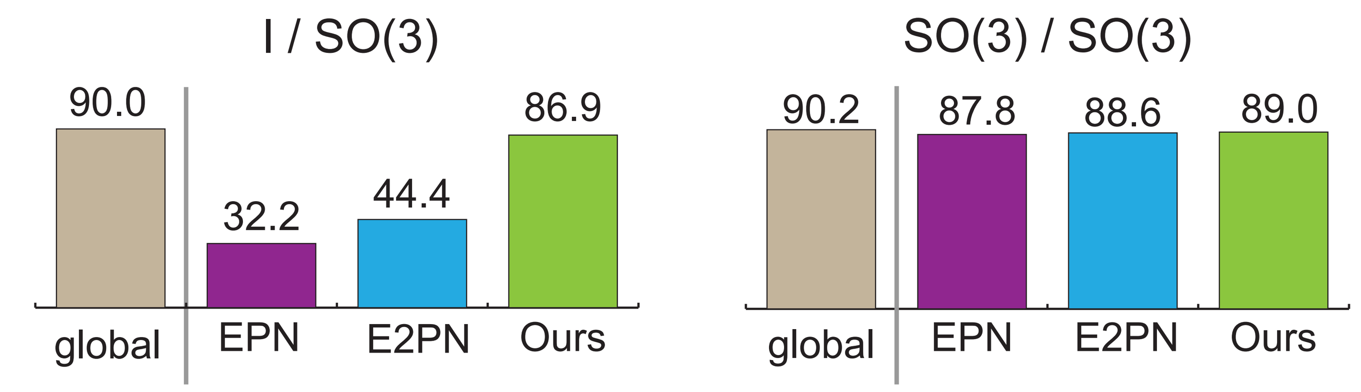


5. Object Classification

ModelNet40: classification accuracy

Method	# samp. train↓ / test→	I / I 1	I / I 2	I / I 4	I / SO(3) 1	I / SO(3) 2	I / SO(3) 4	SO(3) / SO(3) 1	SO(3) / SO(3) 2	SO(3) / SO(3) 4
MC	1	85.4	84.6	83.1	78.8	74.1	70.1	86.5	85.6	84.4
	2	86.2	87.0	87.1	80.3	82.3	82.3	87.1	87.0	87.0
	4	84.2	87.4	87.5	78.4	85.6	86.2	85.4	88.3	88.2
Ours	1	86.9	86.8	86.7	85.5	85.3	85.3	88.7	88.5	88.5
	2	87.9	87.9	87.7	86.6	86.9	86.8	88.9	88.7	88.7
	4	73.2	87.6	87.8	61.4	85.7	86.5	59.7	89.0	88.7
STD			90.7			12.3			87.5	

Global vs. local rotational equivariance (mAcc)

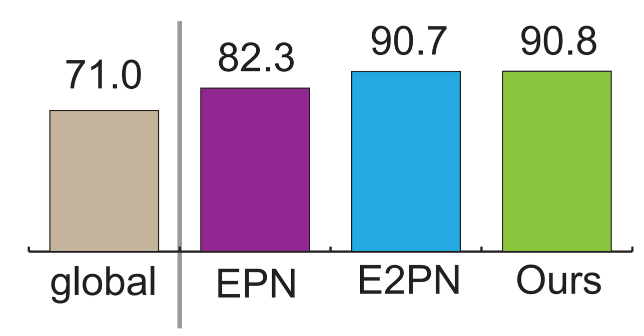


6. Human Body Parts Segmentation

DFAUST: out-of-distribution poses

Method	# samp. train↓ / test→	mAcc 1	mAcc 2	mAcc 4	mIoU 1	mIoU 2	mIoU 4
MC	1	93.1	93.0	92.7	87.7	87.6	87.2
	2	93.8	93.9	93.8	88.8	89.0	88.7
	4	93.4	94.2	94.4	87.9	89.3	89.7
Ours	1	93.8	93.9	93.9	88.9	88.9	89.0
	2	94.3	94.4	94.5	89.7	89.9	89.9
	4	32.6	92.4	95.0	21.6	86.8	90.8
STD			85.3			74.5	

Global vs. local rotational equivariance (mIoU)



7. Memory & Computational Footprint

point-specific grid + stochastic sampling:

SE(3)-equivariant convolutions with negligible computational overhead.

Method	# samp.	Mem. (Mb)↓	FPS↑
STD		37.1	704.2
Ours	1	37.1	581.4
	2	76.9	432.9
	4	165.2	255.8
E2PN		1211.6	45.0
EPN		1636.4	10.2